

Problem Corner Solutions

1.

$$1.2g = .9b$$

$$\frac{1.2g}{.9} = b$$

$$\frac{4}{3}g = b$$

The number of girls must increase by $\frac{1}{3}$.

2.

$$3^5 + 40^2 + 13^2 = 2012 \text{ and } 3^5 + 37^2 + 20^2 = 2012$$

If $b - c$ is a power of three, the 2nd equation is discarded ($40 - 13 = 27 = 3^3$)

$$\text{so } a = 3, b = 40, c = 13 \text{ and } a + b + c = 56$$

3.

Let $\log_2 x = a$ and $\log_2 y = b$. Then $x = 2^a$ and $y = 2^b$.

It follows that $\log_x 2^a = 1$ and $\log_y 2^b = 1$

$$\text{So } a \cdot \log_x 2 = 1 \text{ and } b \cdot \log_y 2 = 1$$

$$\text{and } \log_x 2 = \frac{1}{a} \text{ and } \log_y 2 = \frac{1}{b}$$

Now, $\frac{1}{a} + \frac{1}{b} = \frac{2}{3}$ and a and b are distinct positive integers, so $a = 2$ and $b = 6$

This leaves $x = 2^2 = 4$ and $y = 2^6 = 64$ and $xy = 256$