#1

Since 356xy is a multiple of 9, its digits must sum to a multiple of 9. Further, 356xy has five different digits.

The sum of all five digits already exceeds 9. The sum cannot be 18 either, because x + y would equal 4 which is not possible if all digits are unique. The sum cannot be 36 because x + y would be 22.

If the sum of the digits is 27, then x + y = 13. The only unique addends could be 4 and 9; 5 and 8; and 6 and 7. Since 3, 5, and 6 cannot be used, the only possible addends are 4 and 9.

Since x < y, x = 4 and y = 9. So, 2x + y = 2(4) + 9 = 17.

#2

$$(\log_6 x^4)(\log_x 6)^2 = 2$$

$$(4\log_6 x)(\log_x 6)(\log_x 6) = 2$$

Using change-of-base:

$$\left(\frac{4\log x}{\log 6}\right)\left(\frac{\log 6}{\log x}\right)\left(\frac{\log 6}{\log x}\right) = 2$$

$$\left(\frac{\log x}{\log 6}\right)\left(\frac{\log 6}{\log x}\right)\left(\frac{\log 6}{\log x}\right) = \frac{1}{2}$$

Simplifying common factors:

$$1 \cdot \left(\frac{\log 6}{\log x}\right) = \frac{1}{2}$$

$$\left(\frac{\log 6}{\log x}\right) = \frac{1}{2}$$

$$\log_x 6 = \frac{1}{2}$$

$$x^{\frac{1}{2}} = 6$$

$$x = 36$$

This solution satisfies the inequality $10 < x \leq 50$

#3

Conditional Probability

Let A = {student is a senior}

Let B = {student is a female}

Let C = {student is a math major}

P(student is a senior | student is a female) = P (A | B) = $\frac{P(A \cap B)}{P\ (B)}$ = 0.64

P(student is a senior and a math major | student is a female) = P ((A ∩ C) | B) = $\frac{P(A \cap B\ \cap C)}{P\ (B)}$ = 0.36

P (student is a math major | student is a senior and female) = P (C | (A ∩ B)) = $\frac{P(A \cap B\ \cap C)}{P\ (A\ \cap B)}$

P (C | (A ∩ B)) = $\frac{P(A \cap B\ \cap C)}{P\ (A\ \cap B)} = \frac{0.36 \bullet P\ (B)}{0.64 \bullet P\ (B)} = \frac{0.36}{0.64} = \frac{36}{100} \bullet \frac{100}{64} = \frac{9}{16}$